White Rabbit calibration procedure

version 1.1

Grzegorz Daniluk
CERN BE-CO-HT

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Contents

1 Introduction

This document describes the calibration procedure for White Rabbit devices.

A White Rabbit link established between two devices is characterized by the hardware delays and fiber propagation latencies presented in figure 1.

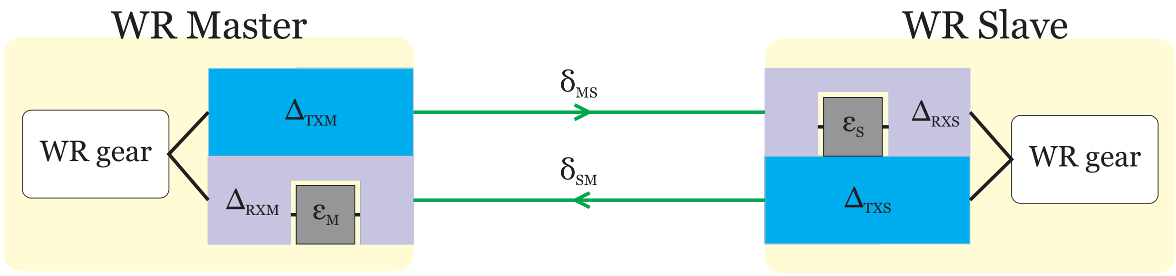

Figure 1: White Rabbit link model

Each WR Master and WR Slave has some constant transmission and reception delays (Δ_{TXM}, Δ_{RXM}, Δ_{TXS}, Δ_{RXS}). They are the summed result of SFP transceiver, PCB trace and electronic component delays as well as the delays inside the FPGA chip. Additional reception delay is also caused on both sides by aligning the recovered clock signal to the inter-symbol boundaries of the data stream. This is called the bitslide value and is marked in figure 1 as ε_M and ε_S.

In addition to hardware delays, packets transmitted in fiber are affected with propagation latencies in both directions (δ_{MS}, δ_{SM}). These fiber propagation latencies are not equal since different light wavelengths are used to communicate simultaneously in both directions using a single fiber.

Based on those parameters the round-trip delay ($delay_{MM}$) is defined as the sum of all delay factors described above:

$$delay_{MM} = \Delta_{TXM} + \Delta_{RXS} + \varepsilon_S + \Delta_{TXS} + \Delta_{RXM} + \varepsilon_M + \delta_{MS} + \delta_{SM} \tag{1}$$

However, since the bitslide values can be easily obtained from WR software, the calibration procedure described in this document use a modified round-trip delay, without ε_M and ε_S:

$$delay'_{MM} = delay_{MM} - \varepsilon_M - \varepsilon_S \tag{2}$$

In the synchronization process, the PTP daemon tries to calculate the current offset between Slave and Master clock. For that it needs a precise estimation of the one-way link delay which, using the described link model, is the sum of four factors:

$$delay_{MS} = \Delta_{TXM} + \delta_{MS} + \Delta_{RXS} + \varepsilon_S \tag{3}$$

The calibration procedure described in this document is required since the values of the hardware transmission and reception delays are not known (Δ_{TXM}, Δ_{RXM}, Δ_{TXS}, Δ_{RXS}). Moreover, fiber cable and each device has unknown asymmetry that prevents from estimating $delay_{MS}$ as half of the round-trip $delay_{MM}$.

2 Definitions

Before going further into the calibration procedure, some initial definitions have to be clarified:

- **WR Device**: any device that supports White Rabbit synchronization over an optical link. This can be e.g. the White Rabbit Switch, the SPEC board, or any custom hardware made for a specific application. A WR Device can run as WR Master or/and WR Slave.
- **WR Calibrator**: any White Rabbit Device selected from the available set that is used as a reference to calibrate the rest of the White Rabbit Devices used in a synchronization system.
- **WR Device TX latency** (Δ_{TX}): WR PTP packet delay from the moment it is timestamped in the transmission circuit inside the FPGA (WR Endpoint module) to the moment when the transmitted packet reaches a fiber link. It is the sum of: transmission delay inside FPGA chip; propagation delay of electronic components and PCB traces; SFP transmitter delay.
- **WR Device RX latency** (Δ_{RX}): WR PTP packet delay from the moment it reaches the receiving SFP module, to the moment it is timestamped in the reception circuit inside the FPGA (WR Endpoint module). It is the sum of: SFP receiver delay; propagation delay of electronic components and PCB traces; reception delay inside FPGA chip.
- **WR Device asymmetry** (2β): the difference between the TX latency Δ_{TX} and the RX latency Δ_{RX}.
- **Fiber round-trip latency** (δ): the total round-trip delay of a fiber link (without the Tx/Rx latencies of the WR Devices). It is the sum of the one-way propagation delays: Master-to-Slave (δ_{MS}) and Slave-to-Master (δ_{SM}).
- **Relative delay coefficient** (α): the parameter describing the asymmetry (the difference between δ_{MS} and δ_{SM}) of the fiber used in a White Rabbit network. It is the relative difference of Master-to-Slave and Slave-to-Master propagation delays ($\alpha = \frac{\delta_{MS}-\delta_{SM}}{\delta_{SM}}$).
- **Round trip delay** ($delay_{MM}$): the total packet delay on a White Rabbit link. It is the sum of communicating parties (Master and Slave) fixed delays (Δ_{TXM}, Δ_{RXM}, Δ_{TXS}, Δ_{RXS}), bitslide values (ε_M, ε_S) and fiber round-trip latency (δ).

3 Equipment requirements

The calibration procedure described in this document makes no assumption on what kind of WR device will be used as a WR Calibrator or what kind of fiber (length, producer) is placed in between WR Devices. However, there are a few general requirements listed below. The first two of them are essential for the Calibrator pre-calibration steps described in section 4.4.

WR Calibration needs:

- two fiber cables having different lengths, one of them is a few kilometers long (to make α coefficient measurement precise), while the other is a few meters long.
- two pieces of a WR Device selected to be the Calibrators: this means two exactly the same WR Devices, having the same PCB design, the same bitstream downloaded into the FPGA and using complementary SFP transceivers (the same producer, matching Tx/Rx WDM wavelengths[1]);
- all WR Devices (i.e. the Calibrator and devices under calibration) capable of producing 1-PPS output from their local clock and providing it through an on-board connector or test point;
- two oscilloscope cables of the same delay (i.e. the same type and length) or different but known delays so that the correction could be made for all oscilloscope measurements described in the document;
- a device of any producer that is able to measure time spans of less than 1ns for measuring 1-PPS skew. This can be an oscilloscope or timer/counter device e.g. Pendulum CNT-91.

[1]Check the list of supported SFP transceivers http://www.ohwr.org/projects/white-rabbit/wiki/SFP

4 Calibration procedures

4.1 Initial state

Initially none of the transmission/reception latencies and asymmetry of White Rabbit devices or fiber parameters are known. Therefore **the values of appropriate parameters should be set to 0 for all nodes and switches which will be calibrated**. In the White Rabbit PTP Core this can be done by modifying the SFP database and setting Δ_{TX}, Δ_{RX} and α values to 0 for all supported transceivers[2]. For the White Rabbit Switch please modify the */wr/etc/sfp_database.conf* and */wr/etc/wrsw_hal.conf* configuration files.

4.2 Reference fiber latency

In this very first step of the calibration procedure the total propagation delay ($\delta = \delta_{MS} + \delta_{SM}$) of a fiber cable is measured. It requires two different fiber cables with LC connectors (i.e. fibers that have different length). For further considerations let's mark them f_1 and f_2. Fiber f_1 can be called helper and is just a few meters long. It is used to measure the parameters of f_2 and in hardware calibration procedures described later. Fiber f_2 is of the same type that will be used in the WR installation. It is a few kilometers long and its parameters will be measured.

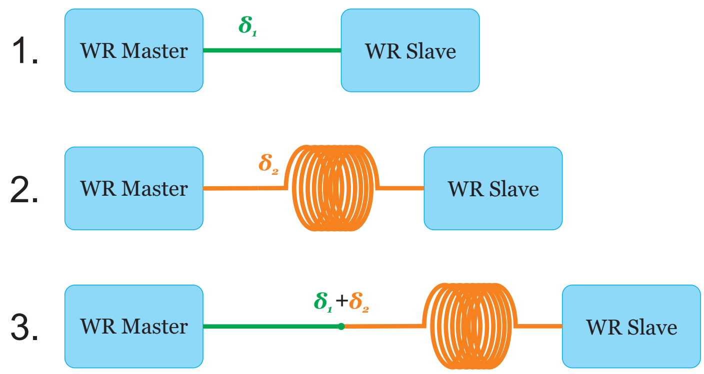

Figure 2: Measuring total fiber propagation latency

1. Take two White Rabbit devices, configure one of them as WR Master and the other as WR Slave.

2. Connect them with fiber f_1 and wait until they become synchronized (fig.2, step 1). You can use the available monitoring software [3] on the Slave device to check when it happens (indicated by the WR servo in the *TRACK_PHASE* state and an offset below 1 ns).

3. Write down the round trip delay ($delay_{MM1}$) reported by the monitoring software and the bitslide values for both Master(ε_{M1}) and Slave(ε_{S1}). Since the fixed delays are initially set

[2] Please refer to the White Rabbit PTP Core manual `http://www.ohwr.org/projects/wr-cores/documents`

[3] *gui* WRPC Shell command for the WR PTP Core, or *wr_mon* program for the WR Switch

to 0, the bitslide values are reported by the monitoring software simply as the reception delays.

4. Connect the same two WR Devices with fiber f_2 (fig.2, step 2), wait again until the WR Slave becomes synchronized and write down the values of $delay_{MM2}$, ε_{M2} and ε_{S2}.

5. Connect fibers f_1 and f_2 together using an LC-LC Adapter and use this link to connect again the same WR Devices together(fig. 2, step 3). Wait until the WR Slave becomes synchronized and write down the values of $delay_{MM3}$, ε_{M3}, ε_{S3}.

6. Subtract the bitslide values to obtain an approximation of the round trip delay, the sum of Master and Slave latencies (Δ_{TXM}, Δ_{RXM}, Δ_{TXS}, Δ_{RXS}) and the fiber round-trip latency:

$$delay'_{MM1} = delay_{MM1} - \varepsilon_{M1} - \varepsilon_{S1} \tag{4}$$
$$delay'_{MM2} = delay_{MM2} - \varepsilon_{M2} - \varepsilon_{S2} \tag{5}$$
$$delay'_{MM3} = delay_{MM3} - \varepsilon_{M3} - \varepsilon_{S3} \tag{6}$$

7. Calculate the latency of the fiber f_1 (δ_1) and f_2 (δ_2):

$$\delta_1 = delay'_{MM3} - delay'_{MM2} \tag{7}$$
$$\delta_2 = delay'_{MM3} - delay'_{MM1} \tag{8}$$

Note: The LC-LC Adapter also has its own unknown latency that introduces an error in the δ_1 and δ_2 calculations. However, this error is a few picoseconds and can be disregarded.

Further mathematical explanation for this calibration step can be found in appendix A.1. Fiber cable f_1 with a round-trip latency δ_1 will be used further to pre-calibrate the selected WR Calibrator.

4.3 Fiber asymmetry

The calibration of a fiber cable is needed to find out its asymmetry coefficient α. In the previous step we managed to calculate the total propagation latency of fibers f_1 and f_2. However, to calibrate any device, the fiber asymmetry has to be known.

The α coefficient used in the White Rabbit protocol to express fiber asymmetry is defined as:

$$\alpha = \frac{\delta_{MS} - \delta_{SM}}{\delta_{SM}} \tag{9}$$

To get its value, you have to make two connections with WR Devices and fibers f_1, f_2 as presented in figure 3.

1. Connect any two WR Devices with a few meters long fiber f_1, set the α value in their configuration to 0, configure one of them as WR Master, and the other as WR Slave. Connect their 1-PPS outputs to an oscilloscope.

   It is important to use oscilloscope cables of the same length and type or cables with known delays so that no additional, unknown latency affects the measurements.

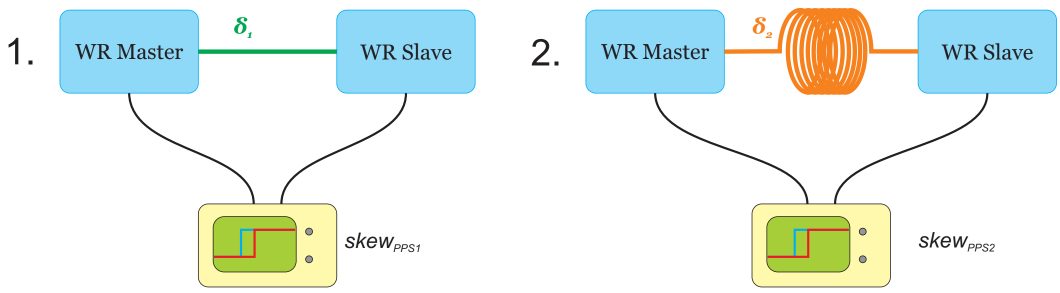

Figure 3: Measuring fiber f_2 asymmetry

2. Use the monitoring software to check when the Slave becomes synchronized to the Master (offset reported by WR PTP daemon below 1 ns and WR servo in the *TRACK_PHASE* state).

3. Measure 1-PPS skew between the Slave and Master in picoseconds by comparing the 1-PPS signals with an oscilloscope ($skew_{PPS1} = t_{PPS_S} - t_{PPS_M}$)

4. Repeat the same procedure using a few kilometers long fiber f_2 to obtain the PPS skew for the second connection ($skew_{PPS2} = t'_{PPS_S} - t'_{PPS_M}$)

5. Calculate the α coefficient using the following equation:

$$\alpha = \frac{2(skew_{PPS2} - skew_{PPS1})}{\frac{1}{2}\delta_2 - (skew_{PPS2} - skew_{PPS1})} \tag{10}$$

6. The White Rabbit Switch uses the α value calculated from the equation above without any conversion. However, the White Rabbit PTP Core requires converting α into a natural number using the formula:

$$\alpha_N = 2^{40}(\frac{\alpha+1}{\alpha+2} - 0.5) \tag{11}$$

7. α or α_N (depending on the WR Device) should be stored into the configuration parameters of a WR Calibrator but also into each Slave device that will use this fiber.

Further mathematical explanation for this calibration step can be found in appendix A.2

Note: Both WR Devices used at CERN: the WR Switch and the WR PTP Core can be used to calculate the α value. However, the 1-PPS signal generated by the WR PTP Core running on the SPEC card with a DIO mezzanine is more jittery than the 1-PPS generated by the WR Switch (check measurement uncertainty considerations in appendix B.2). For most applications the inaccuracy of α measured using two SPEC cards will be negligible. If that isn't the case, the most precise value of the α parameter can be obtained from the measurements described above done with two WR Switches. Alternatively, measurements done with two SPEC cards (or any other hardware based on the SPEC design) can be repeated multiple times to produce an averaged value of 1-PPS skew and a more accurate α.

4.4 Calibrator pre-calibration

Having the parameters of fiber f_1 measured, a WR Calibrator has to be selected among the set of available WR-compatible devices. The only constraint for the selection is having two instances of the same device for the pre-calibration procedure described here. They must have the same PCB layout and the same FPGA bitstream.

To determine what are the transmission and reception latencies (Δ_{TX}, Δ_{RX}) of the WR Calibrator, the connection shown in figure 4 has to be established.

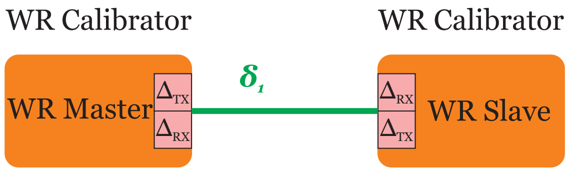

Figure 4: Measuring Calibrator latencies

1. Since both Master and Slave are exactly the same devices (the same FPGA bitstreams, PCB layout, complementary SFPs from the same producer) the sum of their TX and RX latencies is equal:
$$\Delta_{TXM} + \Delta_{RXM} = \Delta_{TXS} + \Delta_{RXS} \tag{12}$$
That means, the total transmission delay caused by each of those devices can be determined by simply dividing by 2 the round trip delay without the RX bitslide of each device and the fiber round-trip latency:
$$\Delta_{TX} + \Delta_{RX} = (delay'_{MM1} - \delta_1)/2 \tag{13}$$

2. Naturally, our Calibrator has some internal asymmetry, which means that $\Delta_{TX} \neq \Delta_{RX}$. There are two ways to determine the exact values of Δ_{TX} and Δ_{RX}. It can be done either by establishing a WR link with a WR Device having a well-known internal asymmetry - which we don't have since all other devices will be calibrated from the WR Calibrator; or taking apart the Calibrator and measuring the latency of PCB traces, electronic components, SFP transceivers - that is possible, not trivial at all, and raises many other problems like measuring the latency between the SFP electrical input and its optical output, or measuring the latency inside the FPGA chip. Therefore at this point we set up a convention, that the WR Calibrator has no asymmetry:
$$\Delta_{TX} = \Delta_{RX} \tag{14}$$
That means, the parameters describing the transmission and reception delays of the WR Calibrator should be set in the device configuration to:
$$\Delta_{TX} = \Delta_{RX} = (delay'_{MM1} - \delta_1)/4 \tag{15}$$
Note: Don't worry, assuming that the WR Calibrator has no asymmetry does not affect the calibration quality. It is taken into account later during the actual calibration of each

unknown WR Device. It also does not affect the WR PTP protocol when a link is established between two WR Calibrators, since the daemon only uses the precise value of the one-way, Master to Slave link delay (where the TX latency of the Master and RX latency of the Slave are summed together).

3. Configure your WR Calibrator with the calculated Δ_{TX} and Δ_{RX} values - modify the SFP database for the WR PTP Core, or the */wr/etc/wrsw_hal.conf* file for the WR Switch.

4.5 WR Device calibration

Having the WR Calibrator selected among the available WR Devices, and the fiber cable used in the previous steps (with known α coefficient and total propagation delay δ), you can start calibrating all other devices that will later create a White Rabbit Network (fig.5).

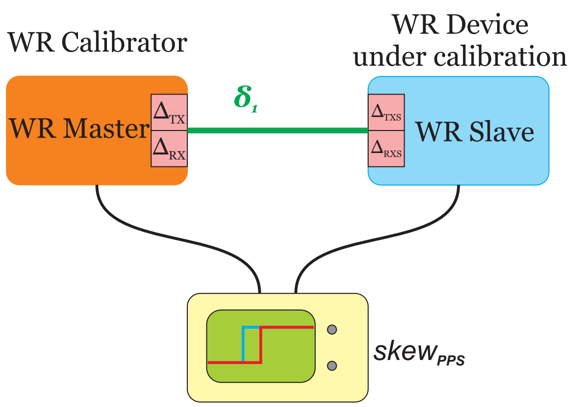

Figure 5: WR Device calibration with WR Calibrator and known fiber f_1

1. Connect your Calibrator to an unknown WR Device using the fiber with the known α measured earlier (4.3). Remember to use an appropriate SFP transceiver on each side depending on whether your device under calibration is supposed to run as a WR Master or a WR Slave[4]. If you want to have the flexibility of selecting Master/Slave mode later through device configuration, the calibration procedure described below should be performed twice - with the SFP that will be used in Master, and the SFP that will be used in Slave mode. In further steps of this procedure the assumption is made that the WR Device being calibrated runs in WR Slave mode.

2. Run the monitoring software on the Slave node and write down the round-trip delay ($delay_{MM}$) and fixed delays of both Master and Slave (Δ_{TXM}, Δ_{RXM}, Δ_{TXS}, Δ_{RXS}). The transmission delay for Slave is reported to be 0, while the reception delay may be non-zero. It is the current RX bitslide value ε_S.

[4]There is a wiki page describing which wavelength should be used on each side: `http://www.ohwr.org/projects/white-rabbit/wiki/SFP`

3. Calculate the average (coarse) transmission and reception delays for the Slave device:

$$\Delta'_{TXS} = \Delta'_{RXS} = \frac{1}{2}\Delta_S = \frac{1}{2}(delay_{MM} - \Delta_{TXM} - \Delta_{RXM} - \varepsilon_S - \delta_1) \tag{16}$$

4. Write the Δ_{TXS} and Δ_{RXS} delays to the configuration of your device, and restart its PTP daemon so that it synchronizes again using the new values.

5. At this point your WR Device knows the coarse values of the transmission and reception delays. You can connect the 1-PPS signals generated by the calibrator and the device to an oscilloscope to observe that there is still some offset of (probably) a few nanoseconds between the Slave and Master even if the PTP daemon reports it is close to 0. That is because the transmission and reception delays are not equal (as the coarse values above). This asymmetry has to be measured and used to correct the coarse values of Δ_{TXS} and Δ_{RXS}.

6. Measure the Slave to Master offset by comparing the 1-PPS skew with the oscilloscope. This is the correction value that has to be subtracted from the coarse Δ_{TX} and added to Δ_{RX} to compensate the hardware asymmetry:

$$\Delta_{TXS} = \frac{1}{2}\Delta_S - skew_{PPS} \tag{17}$$
$$\Delta_{RXS} = \frac{1}{2}\Delta_S + skew_{PPS} \tag{18}$$

   Note: The asymmetry measured in this stage of calibration is in fact the sum of WR Device and WR Calibrator asymmetry. However, since both transmission and reception delays are modified with this value, the component for WR Calibrator asymmetry will cancel when connecting two WR Devices calibrated to the same Calibrator, see A.3 for a mathematical proof.

7. After putting the new delay values in the configuration of the WR Device, the PTP daemon can be restarted and this time the device will synchronize to the Calibrator with an offset below 1ns.

8. This procedure has to be repeated for all other WR Devices (WR Masters and WR Slaves). When you want to calibrate a WR Switch, this calibration procedure has to be performed for each WR port of the device.

5 Measuring already deployed fiber

Most of the methods described in section 4 require making a measurement of 1-PPS signals produced by WR Devices. It's fairly easy to compare them when you have a roll of fiber and the devices being calibrated lying on a desk in the laboratory. However, sometimes the fiber infrastructure which will be used in a WR network is already installed. WR Devices can still be collected in a lab for calibration, but measuring the α parameter of a buried fiber is not as straightforward as in section 4.3. This section proposes a way of measuring 1-PPS skew when the fiber cable is already installed and you don't have both ends next to each other. The measurements made this way can be used as a substitute for the simple oscilloscope measurements in section 4.

5.1 Measurement with a loop-back fiber

This method is based on a fiber delay calibration procedure used in the CERN Neutrinos to Gran Sasso project [1]. It requires:

- an additional fiber which will be used to transmit the 1-PPS signal produced by a distant WR Device to the place where it can be compared locally with the other WR Device using an oscilloscope
- two oscilloscope cables of the same type - having equal latency or a known latency difference which can be taken into account and compensated
- one pair of optical transmitter and receiver which will be used to convert the 1-PPS electric signal into a light impulse sent through the loop-back fiber. The transmitter and receiver must have constant transmission/reception delays that don't vary on each power cycle.

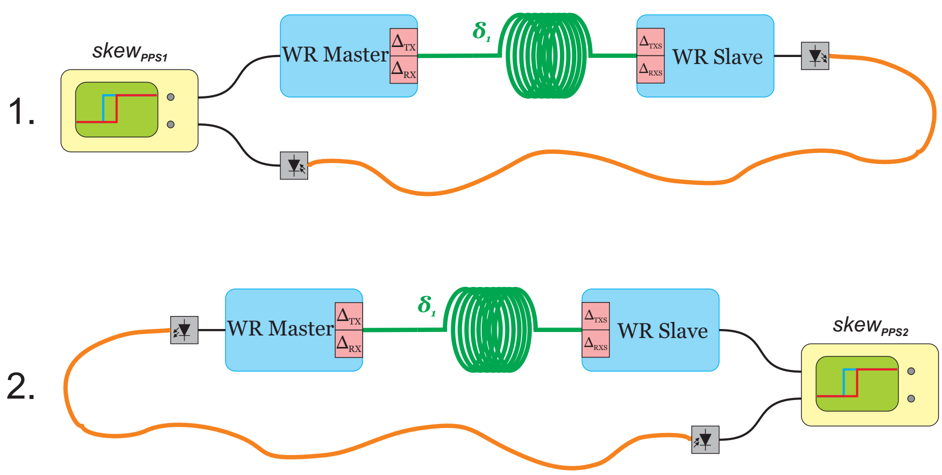

Figure 6: Measuring 1-PPS offset using a loop-back fiber

Measurement procedure:

1. When a link is established between the WR Master and the WR Slave, connect the optical transmitter to the 1-PPS signal produced by the WR Slave and the loop-back fiber. On the other side of the fiber connect the optical receiver (fig.6, step 1).

2. Measure the 1-PPS skew ($skew_{PPS1}$) between the Slave (1-PPS transmitted through the loop-back fiber) and the Master (1-PPS directly from the node) using an oscilloscope.

3. Swap the optical transmitter and receiver so that you'll now transfer the 1-PPS signal generated from the WR Master using the same loop-back fiber to the WR Slave side (fig.6, step 2).

4. Do the same 1-PPS skew measurement between the Slave and Master, but this time on the WR Slave side (the 1-PPS comes directly from the WR Slave node and the one from the WR Master is transmitted through the loop-back fiber). This way you will obtain the value $skew_{PPS2}$.

5. Calculate the actual skew between the WR Master and WR Slave using the equation:

$$skew_{PPS} = \frac{1}{2}(skew_{PPS1} + skew_{PPS2}) \tag{19}$$

The skew value calculated that way can be used in any equation from section 4. However, please remember that you will also need the latency of the fiber (δ_1 in figure 6). That means, you would have to start the calibration with two WR Devices on your desk, connected with a short (few meters long) link (procedure 4.2) and later use these two WR Devices with the already-installed link.

6 Recovering the calibrator

In a real life, when a WR network operates for a few years, the WR Device selected to be a calibrator might become broken. The lack of a WR Calibrator makes it impossible to connect new devices without recalibrating the whole WR network. This section presents a method for obtaining the Δ_{TX}, Δ_{RX} delays for a WR Device so that it can be used as the new WR Calibrator for an already-deployed network.

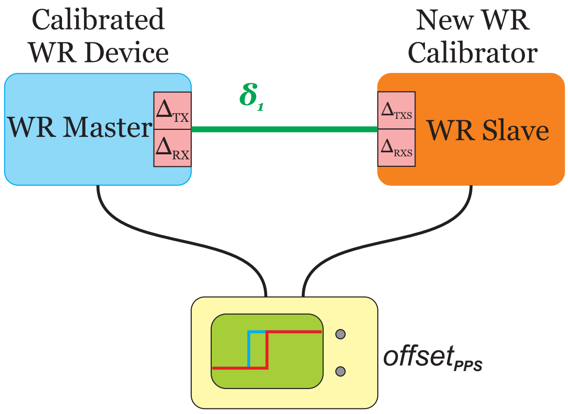

Figure 7: Calibrating a new WR Calibrator for an already-existing WR network

1. Initially please set the transmission and reception delays (Δ_{TXS}, Δ_{RXS}) of the new WR Calibrator to 0, but α has to be set to the correct value for fiber f_1.

2. Connect your new WR Calibrator to one of the WR Devices originally calibrated to the primary WR Calibrator (fig.7). Use a short (few meters) fiber of known latency and α coefficient or measure it first according to the instructions in sections 4.2 and 4.3. Set the WR Calibrator to Slave, and the WR Device to Master mode.

3. Run the monitoring software on the Slave node (*wr_mon* for the WR Switch or *gui* for the WR PTP Core). Write down the round-trip delay and fixed delays of both Master and Slave. The transmission delay of the Slave (Δ_{TXS}) should be 0, but its reception delay (Δ_{RXS}) will be equal to the RX bitslide value ε_S. The reception delay of the Master (Δ_{RXM}) already includes the bitslide value for this device.

4. Calculate the average (coarse) transmission and reception delay for the new Calibrator using the values read from the monitoring software and the latency of the fiber (δ_1):

$$\Delta'_{TXS} = \Delta'_{RXS} = \frac{1}{2}\Delta_S = \frac{1}{2}(delay_{MM} - \Delta_{TXM} - \Delta_{RXM} - \Delta_{RXS} - \delta_1) \qquad (20)$$

5. Write the Δ_{TXS} and Δ_{RXS} delays to the configuration of the new WR Calibrator, restart the device, and let it synchronize again with the new values.

6. Measure the Slave to Master offset by comparing 1-PPS signals with an oscilloscope. The 1-PPS skew ($skew_{PPS}$) is used as a correction value for the coarse transmission/reception delays:

$$\Delta_{TXS} = \frac{1}{2}\Delta_S - skew_{PPS} \tag{21}$$

$$\Delta_{RXS} = \frac{1}{2}\Delta_S + skew_{PPS} \tag{22}$$

7. Update the configuration of the new WR Calibrator to replace the coarse delays with Δ_{TXS}, Δ_{RXS}.

8. The new WR Calibrator can be used to calibrate any other WR Device that is supposed to be connected to the existing WR network.

   Note: Please be aware that the measurement errors accumulate. It might become an issue if the new calibrator uses values recovered with a WR Device that was already calibrated to a recovered calibrator. The longer this chain is, the more inaccuracy you should expect from the calibration procedure. To minimize this effect, it's better to recover the values for the new WR Calibrator using only devices that were calibrated to the primary WR Calibrator for a given network.

A Mathematical proofs

A.1 Reference fiber latency

The same pair of WR Devices is used for all three connections in this procedure. That is why, when considering round-trip delay after subtracting the bitslide values, the transmission and reception delays of both devices are summed together and remain constant for fiber f_1, f_2, $f_1 + f_2$:

$$\Delta = \Delta_{TXM} + \Delta_{RXM} + \Delta_{TXS} + \Delta_{RXS} \tag{23}$$

When two fibers f_1, f_2 are joined together the fiber latency for this connection will be the sum of δ_1 and δ_2. After eliminating the bitslide value, the remaining part of a round trip delay consists of the following elements:

$$delay'_{MM1} = \Delta + \delta_1 \tag{24}$$

$$delay'_{MM2} = \Delta + \delta_2 \tag{25}$$

$$delay'_{MM3} = \Delta + \delta_1 + \delta_2 \tag{26}$$

This equation system has three unknowns and after solving gives the formulas for the round-trip fiber latencies:

$$\delta_1 = delay'_{MM3} - delay'_{MM2} \tag{27}$$

$$\delta_2 = delay'_{MM3} - delay'_{MM1} \tag{28}$$

A.2 Fiber asymmetry

In this step of the calibration procedure two WR connections with the same pair of devices are established. For each of them the offset between the WR Slave and WR Master is calculated by the WR PTP software as:

$$offset_{MS} = t_1 - t_2 + delay_{MS} \tag{29}$$

where $delay_{MS}$ is an estimated one-way link delay. When α is initially equal to 0, $delay_{MS}$ is estimated as half of the round-trip delay, which results in a distorted offset between the two devices:

$$offset'_{MS} = t_1 - t_2 + \frac{1}{2} delay_{MM} \tag{30}$$

Then $skew_{PPS}$ measured with an oscilloscope is equal to the uncompensated link asymmetry (the sum of the fiber asymmetry and hardware asymmetry):

$$skew_{PPS1} = offset_{MS1} - offset'_{MS1} = delay_{MS1} - \frac{1}{2} delay_{MM1} \tag{31}$$

$$skew_{PPS2} = offset_{MS2} - offset'_{MS2} = delay_{MS2} - \frac{1}{2} delay_{MM2} \tag{32}$$

We also know what factors build up the round trip delays and one-way delays for both connections. Please notice since the same pair of the devices is used in both cases, fixed hardware delays stay the same:

$$delay_{MM1} = \Delta + \delta_1 \tag{33}$$
$$delay_{MM2} = \Delta + \delta_2 \tag{34}$$
$$delay_{MS1} = \Delta_{TXM} + \Delta_{RXS} + \delta_{MS1} \tag{35}$$
$$delay_{MS2} = \Delta_{TXM} + \Delta_{RXS} + \delta_{MS2} \tag{36}$$
$$\delta_1 = \delta_{MS1} + \delta_{SM1} \tag{37}$$
$$\delta_2 = \delta_{MS2} + \delta_{SM2} \tag{38}$$

Using the formulas above, equations 31 and 32 can be expanded:

$$skew_{PPS1} = \Delta_{TXM} + \Delta_{RXS} + \delta_{MS1} - \frac{1}{2}\Delta - \frac{1}{2}\delta_1 \tag{39}$$
$$skew_{PPS2} = \Delta_{TXM} + \Delta_{RXS} + \delta_{MS2} - \frac{1}{2}\Delta - \frac{1}{2}\delta_2 \tag{40}$$

Subtracting the two skew measurements eliminates any asymmetry due to fixed hardware delays:

$$\begin{aligned} skew_{PPS} &= skew_{PPS2} - skew_{PPS1} && (41)\\ &= \Delta_{TXM} + \Delta_{RXS} - \Delta_{TXM} - \Delta_{RXS} + \delta_{MS2} - \delta_{MS1} - \frac{1}{2}\Delta + \frac{1}{2}\Delta - \frac{1}{2}\delta_2 + \frac{1}{2}\delta_1 \\ &= \delta_{MS2} - \delta_{MS1} - \frac{1}{2}\delta_2 + \frac{1}{2}\delta_1 && (42) \end{aligned}$$

However, if fiber f_1 is just a few meters long, then its asymmetry is negligible. That means its one-way Master-to-Slave latency equals half of the total fiber latency:

$$\delta_{MS1} = \frac{1}{2}\delta_1 \tag{43}$$

This results in a simplified formula describing $skew_{PPS}$:

$$\begin{aligned} skew_{PPS} &= \delta_{MS2} - \frac{1}{2}\delta_2 = \delta_{MS2} - \frac{1}{2}\delta_{MS2} - \frac{1}{2}\delta_{SM2} \\ &= \frac{1}{2}(\delta_{MS2} - \delta_{SM2}) && (44) \end{aligned}$$

Having in mind that $\alpha = \frac{\delta_{MS} - \delta_{SM}}{\delta_{SM}}$, using the already known value of the f_2 round-trip latency δ_2 and equations 41, 44 we get the expression for α used in the calibration procedure:

$$\alpha = \frac{2(skew_{PPS2} - skew_{PPS1})}{\frac{1}{2}\delta_2 - (skew_{PPS2} - skew_{PPS1})} \tag{45}$$

A.3 WR Device calibration

After the WR PTP daemon on a Slave device is synchronized to Master, the $skew_{PPS}$ observed on an oscilloscope can be treated as an error of a clock correction on the Slave side:

$$corr = corr_{ideal} - skew_{PPS} \tag{46}$$

The correction value that should be applied to the Slave clock by the daemon ($corr_{ideal}$) is calculated based on timestamps and a $delay_{MS}$ estimation:

$$corr_{ideal} = t_1 - t_2 + delay_{MS_{ideal}} \tag{47}$$

The one-way delay is the sum of the fiber latency, Master transmission delay and Slave reception delay:

$$delay_{MS_{ideal}} = \frac{1+\alpha}{2+\alpha}(delay_{MM} - \Delta) + \Delta_{TXM} + \Delta_{RXS} \tag{48}$$

However, the Slave reception delay used by the daemon is the result of the first 4 steps of the procedure in 4.5 ($\frac{1}{2}\Delta_S$). That means, it has to be corrected by an asymmetry coefficient β to get the right value that produces $corr_{ideal}$ above:

$$\Delta_{RXS} = \frac{1}{2}\Delta_S + \beta \tag{49}$$

The round-trip delay value and the sum of hardware delays are fixed, which means the same asymmetry factor has to be subtracted from the Slave transmission delay to preserve those sums:

$$\Delta_{TXS} = \frac{1}{2}\Delta_S - \beta \tag{50}$$

Taking it back to equation 48 we get:

$$delay_{MS_{ideal}} = \frac{1+\alpha}{2+\alpha}(delay_{MM} - \Delta) + \Delta_{TXM} + \frac{1}{2}\Delta_S + \beta \tag{51}$$

However, the Master to Slave delay calculated by the daemon using the values without the asymmetry taken into account is:

$$delay_{MS} = \frac{1+\alpha}{2+\alpha}(delay_{MM} - \Delta) + \Delta_{TXM} + \frac{1}{2}\Delta_S \tag{52}$$

So the correction value for the reception asymmetry is also the difference between the $delay_{MS}$ estimations:

$$delay_{MS_{ideal}} = delay_{MS} + \beta \tag{53}$$

Putting this back into the equation for $corr_{ideal}$:

$$corr_{ideal} = t_1 - t_2 + delay_{MS} + \beta \tag{54}$$

Please remember though, $t_1 - t_2 + delay_{MS}$ is in fact the correction value ($corr$) derived from the coarse (without asymmetry) Slave delays:

$$corr_{ideal} = corr + \beta \tag{55}$$

Comparing the equation above with 46:

$$\beta = skew_{PPS} \tag{56}$$

That means, the difference between 1-PPS signals observed on the oscilloscope has to be used as the correction factor for the coarse delays of the Slave device.

The asymmetry of each calibrated Tx/Rx delay is set to compensate also the asymmetry of the WR Calibrator. Equations 49 and 50 can be expanded to show the components of asymmetry β of two WR Devices calibrated to the same WR Calibrator (where β_C is the calibrator asymmetry and β_1, β_2 are the internal asymmetries of each device):

$$\Delta_{TX1} = \frac{1}{2}\Delta_1 - \beta_{C1} = \frac{1}{2}\Delta_1 - \beta_1 + \beta_C \tag{57}$$

$$\Delta_{RX1} = \frac{1}{2}\Delta_1 + \beta_{C1} = \frac{1}{2}\Delta_1 + \beta_1 - \beta_C \tag{58}$$

$$\Delta_{TX2} = \frac{1}{2}\Delta_2 - \beta_{C2} = \frac{1}{2}\Delta_2 - \beta_2 + \beta_C \tag{59}$$

$$\Delta_{RX2} = \frac{1}{2}\Delta_2 + \beta_{C2} = \frac{1}{2}\Delta_2 + \beta_2 - \beta_C \tag{60}$$

After connecting those two WR Devices together, the transmission circuits of each one communicate with the reception circuits of the other, resulting in a one-way link delay (without fiber propagation latency):

$$\Delta_{1-2} = \Delta_{TX1} + \Delta_{RX2} = \frac{1}{2}\Delta_1 - \beta_1 + \beta_C + \frac{1}{2}\Delta_2 + \beta_2 - \beta_C = (\frac{1}{2}\Delta_1 - \beta_1) + (\frac{1}{2}\Delta_2 + \beta_2) \tag{61}$$

$$\Delta_{2-1} = \Delta_{TX2} + \Delta_{RX1} = \frac{1}{2}\Delta_2 - \beta_2 + \beta_C + \frac{1}{2}\Delta_1 + \beta_1 - \beta_C = (\frac{1}{2}\Delta_2 - \beta_2) + (\frac{1}{2}\Delta_1 + \beta_1) \tag{62}$$

This proves that devices which have been calibrated using the same WR Calibrator can use the asymmetries found during the calibration process to synchronize one another.

A.4 Measurement with a loop-back fiber

For both measurements the same loop-back fiber, optical transmitter and optical receiver are used. There is also a requirement in the measurement procedure (section 5.1) saying that both transmitter and receiver should have a constant delay that doesn't vary for each connection. That means, for both steps, the loop-back link has some unknown latency δ_L.

In the first case, the 1-PPS skew measured on the WR Master side can be represented with the formula:

$$skew_{PPS1} = t_{PPSM1} - (t_{PPSS1} + \delta_L) \tag{63}$$

where t_{PPSM1} is a WR Master absolute time of 1-PPS generation, t_{PPSS1} is a WR Slave absolute time of 1-PPS generation. The latency of the loop-back fiber δ_L is added to t_{PPSS1}, because in the first step the Slave 1-PPS signal observed on the WR Master side is delayed by δ_L picoseconds.

In the second step, the situation is reversed. The measurement is made on the WR Slave side, which means the 1-PPS generated from the WR Master is observed δ_L picoseconds later:

$$skew_{PPS2} = (t_{PPSM2} + \delta_L) - t_{PPSS2} \tag{64}$$

The actual $skew_{PPS}$ that we want to measure within this procedure is the difference between the absolute time of the 1-PPS generation on Master and Slave:

$$skew_{PPS} = t_{PPSM1} - t_{PPSS1} = t_{PPSM2} - t_{PPSS2} \tag{65}$$

Of course we can make those subtractions equal only because the measurement in both cases is done when WR Master and WR Slave are synchronized. Now, putting together equations 63, 64 and 65 the following system of equations with two unknowns is produced:

$$skew_{PPS1} = skew_{PPS} - \delta_L \tag{66}$$
$$skew_{PPS2} = skew_{PPS} + \delta_L \tag{67}$$

Solving it creates the final formula to calculate the 1-PPS skew between the WR Master and the WR Slave:

$$skew_{PPS} = \frac{1}{2}(skew_{PPS1} + skew_{PPS2}) \tag{68}$$

A.5 Recovering the calibrator

The new WR Calibrator has unknown transmission and reception delays as any other, uncalibrated WR Device. We represent them using the mean (coarse) delay (Δ_{C2}) and the asymmetry factor (β_{C2}):

$$\Delta_{TXC2} = \frac{1}{2}\Delta_{C2} - \beta_{C2} \tag{69}$$
$$\Delta_{RXC2} = \frac{1}{2}\Delta_{C2} + \beta_{C2} \tag{70}$$

We already know from the previous sections that a WR Device (D1) calibrated to the primary calibrator (C1) compensates its own asymmetry but also the asymmetry of the WR Calibrator:

$$\Delta_{TXD1} = \frac{1}{2}\Delta_{D1} - \beta_{D1} + \beta_{C1} \tag{71}$$
$$\Delta_{RXD1} = \frac{1}{2}\Delta_{D1} + \beta_{D1} - \beta_{C1} \tag{72}$$

In an ideal case, when each WR Device knows its delays, the Master-to-Slave (one-way) delay without the fiber propagation latency would be:

$$\Delta_{D1-C2_{ideal}} = \Delta_{TXD1_{ideal}} + \Delta_{RXC2_{ideal}} = \frac{1}{2}\Delta_{D1} - \beta_{D1} + \frac{1}{2}\Delta_{C2} + \beta_{C2} \tag{73}$$

On the other hand, since the WR Device *D1* compensates also the asymmetry of the primary calibrator *C1* and initially β_{C2} is unknown (set to 0), the actual fixed delay for *D1-C2* connection is:

$$\Delta_{D1-C2} = \frac{1}{2}\Delta_{D1} - \beta_{D1} + \beta_{C1} + \frac{1}{2}\Delta_{C2} \tag{74}$$

Comparing equations 74 and 73 it can be noticed that the factor β_{C1} partially compensates the asymmetry of the new calibrator *C2*. The uncompensated part:

$$\beta'_{C2} = \beta_{C2} - \beta_{C1} \tag{75}$$

produces an additional skew of the 1-PPS signals in the same way as the uncompensated asymmetry of the WR Device in section A.3:

$$skew_{PPS} = \beta_{C2} - \beta_{C1} \tag{76}$$

This remaining asymmetry of the *D1-C2* connection is compensated in the calibration procedure by using the 1-PPS skew as the correction factor. Then, the transmission and reception delays of the new calibrator *C2* are presented in the equations:

$$\Delta_{TXC2} = \frac{1}{2}\Delta_{C2} - skew_{PPS} = \frac{1}{2}\Delta_{C2} - \beta_{C2} + \beta_{C1} \tag{77}$$

$$\Delta_{RXC2} = \frac{1}{2}\Delta_{C2} + skew_{PPS} = \frac{1}{2}\Delta_{C2} + \beta_{C2} - \beta_{C1} \tag{78}$$

Each of them has the asymmetry factor β_{C2} reduced by β_{C1} so that the actual hardware asymmetry is reduced only partially. The remaining, uncompensated part equals the asymmetry of the primary calibrator *C1*, so that the new calibrator *C2* behaves for all practical purposes as the old calibrator *C1*.

A.6 1-PPS skew measurement

Reading proposed calibration procedure one can start wondering what is the influence of 1-PPS propagation time - from the inside of FPGA to the physical connector - on the 1-PPS skew measurement.

Let's consider one more time two WR Devices (*D1*, *D2*) being calibrated to the calibrator *C*. This time we take into account the 1-PPS propagation delay from the inside of FPGA to the physical connector where we take it for skew measurement (figure 8). Those delays are marked τ_C, τ_1, τ_2 for the calibrator, device under calibration 1 and device under calibration 2. Therefore e.g. 1-PPS signal generated inside the FPGA of the WR Calibrator at time t_{CPPS} is

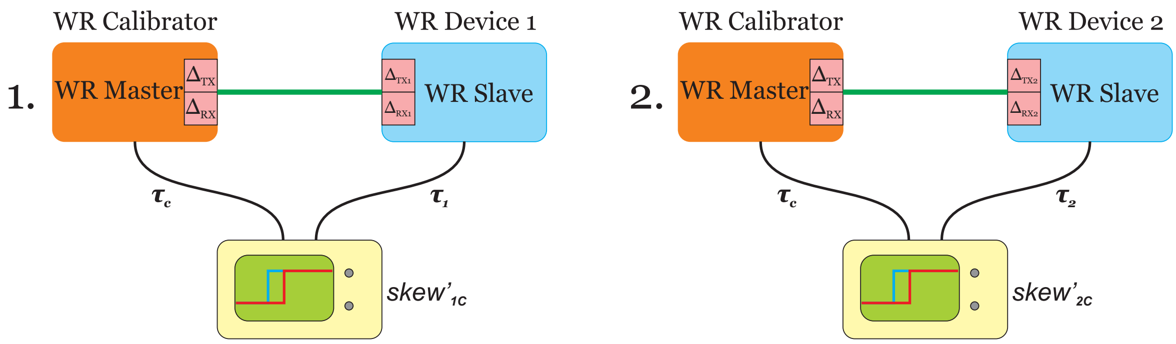

Figure 8: Calibration with 1-PPS delays taken into account

observed on the oscilloscope at time $t_{CPPS} + \tau_C$. Taking this into account, our skew measured in section 4.5 can be expanded as:

$$\begin{aligned} skew'_{1C} &= (t_{1PPS} + \tau_1) - (t_{CPPS} + \tau_C) \\ &= (t_{1PPS} - t_{CPPS}) + (\tau_1 - \tau_C) \\ &= skew_{1C} + (\tau_1 - \tau_C) \end{aligned} \tag{79}$$

According to the calibration procedure we apply the measured skew (in our case $skew'_{1C}$) as an asymmetry factor (β_{C1} in section A.3) to calculate fixed hardware delays Δ_{TX}, Δ_{RX} for the device under calibration. Thus, our asymmetry factor also contains the difference in 1-PPS propagation times:

$$\beta'_{C1} = \beta_{C1} + (\tau_1 - \tau_C) \tag{80}$$

As a consequence, fixed transmission and reception delays for device D_1 calculated from the coarse delay and asymmetry factor β'_{C1} will also contain 1-PPS propagation times:

$$\Delta'_{TX1} = \frac{1}{2}\Delta_1 - \beta'_{C1} = \frac{1}{2}\Delta_1 - \beta_{C1} - (\tau_1 - \tau_C) = \Delta_{TX1} - (\tau_1 - \tau_C) \tag{81}$$

$$\Delta'_{RX1} = \frac{1}{2}\Delta_1 + \beta'_{C1} = \frac{1}{2}\Delta_1 + \beta_{C1} + (\tau_1 - \tau_C) = \Delta_{RX1} + (\tau_1 - \tau_C) \tag{82}$$

By analogy we get the same result for device D_2 calibration:

$$\Delta'_{TX2} = \frac{1}{2}\Delta_2 - \beta'_{C2} = \frac{1}{2}\Delta_2 - \beta_{C2} - (\tau_2 - \tau_C) = \Delta_{TX2} - (\tau_2 - \tau_C) \tag{83}$$

$$\Delta'_{RX2} = \frac{1}{2}\Delta_2 + \beta'_{C2} = \frac{1}{2}\Delta_2 + \beta_{C2} + (\tau_2 - \tau_C) = \Delta_{RX2} + (\tau_2 - \tau_C) \tag{84}$$

We can see that after performing the calibration procedure, both of these devices have their fixed hardware delays distorted by the difference in 1-PPS propagation delay of the device and the calibrator.

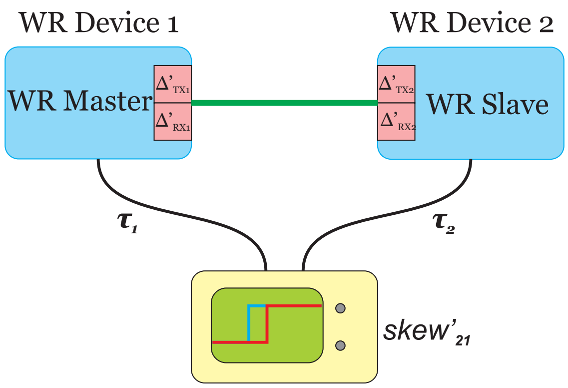

Figure 9: Synchronization of WR Devices calibrated to the same calibrator

When we connect two devices together and let them synchronize (figure 9), the propagation delay of calibrator's PPS signal gets canceled in the one-way delay calculation:

$$delay'_{MS} = \delta_{MS} + \Delta'_{TX1} + \Delta'_{RX2} = \delta_{MS} + (\Delta_{TX1} - \tau_1 + \tau_C) + (\Delta_{RX2} + \tau_2 - \tau_C) \tag{85}$$

$$delay'_{MS} = delay_{MS} + (\tau_2 - \tau_1) \tag{86}$$

Now, the one-way delay is distorted only by the difference between D_2 and D_1 1-PPS propagation delays. Having in mind the formula for a correction factor applied on the Slave side ($corr_{ideal}$ in section A.3) we can see that in our case the distortion of $delay_{MS}$ directly affects $corr_{ideal}$ as well:

$$corr = corr_{ideal} + (\tau_2 - \tau_1) \tag{87}$$

The change in the *corr* value shifts the timescale of the slave device ahead by $(\tau_2 - \tau_1)$ so for this connection every 1-PPS pulse from D_2 is generated earlier than it should be in the ideal case:

$$t'_{2PPS} = t_{2PPS} - (\tau_2 - \tau_1) \tag{88}$$

$skew'_{12}$ between D_1 and D_2 measured with the oscilloscope is:

$$skew'_{21} = (t'_{2PPS} + \tau_2) - (t_{1PPS} + \tau_1) = t_{2PPS} - \tau_2 + \tau_1 + \tau_2 - t_{1PPS} - \tau_1 \tag{89}$$

$$skew'_{21} = t_{2PPS} - t_{1PPS} \tag{90}$$

This shows that the difference between 1-PPS propagation delays causes Slave device to have its internal time shifted to compensate for this difference when two devices (calibrated earlier to the same calibrator) are connected together. Therefore when these devices are synchronized, their 1-PPS signals will be aligned and the difference in their propagation times is properly compensated.

1-PPS socket is our calibration reference plane. One can move the reference plane to any other point, also to the inside of the FPGA. However, this requires the knowledge of precise 1-PPS delay value to be taken into account in the PPS skew measurements.

B Measurement errors estimation

Notation used in this section is based on the *BIPM* document on uncertainty in measurements[2]:

- q_k — single sample of the measured quantity q
- $\bar{q}$ — arithmetic mean calculated from N independent observations of quantity q: $\bar{q} = \frac{1}{N}\sum_{k=1}^{N} q_k$
- $s^2(q_k)$ — experimental variance of the q observation calculated using N samples: $s^2(q_k) = \frac{1}{N-1}\sum_{j=1}^{N}(q_j - \bar{q})^2$
- $s^2(\bar{q})$ — experimental variance of the mean value of q: $s^2(\bar{q}) = \frac{s^2(q_k)}{N}$
- $s(\bar{q})$ — experimental standard deviation of the mean
- $u^2(q)$ — uncertainty of the quantity q estimation: $u^2(q) = s^2(\bar{q})$
- $u_c^2(y)$ — combined standard uncertainty of the value y determined from N other quantities $y = f(x_1, x_2, ..., x_N)$: $u_c^2(y) = \sum_{n=1}^{N}\left(\frac{\partial f}{\partial x_n}\right)^2 u^2(x_i)$

B.1 Fiber latency measurement

In section 4.2 the latency of the fiber is calculated based on the round trip delays after subtracting the bitslides ε_M, ε_S ($delay'_{MM}$):

$$\delta_1 = delay'_{MM3} - delay'_{MM2} \tag{91}$$
$$\delta_2 = delay'_{MM3} - delay'_{MM1} \tag{92}$$

WR Devices currently used know the precise value of a bitslide, which comes directly from the GTP/GTX SerDes. The inaccuracy of the bitslide can be ignored, and the uncertainty of $delay_{MM}$ treated as the uncertainty of $delay'_{MM}$.

Note: If an alternative implementation needs to perform an on-line calibration to measure the delays of an external Ethernet PHY, the uncertainty of such measurement would have to be added to the considerations below.

The combined standard uncertainty of the fiber latency measurement can be estimated with the formula:

$$\begin{aligned} u_c^2(\delta_1) &= \left(\frac{\partial \delta_1}{\partial delay_{MM3}}\right)^2 u^2(delay_{MM3}) + \left(\frac{\partial \delta_1}{\partial delay_{MM2}}\right)^2 u^2(delay_{MM2}) \\ &= u^2(delay_{MM3}) + u^2(delay_{MM2}) \end{aligned} \tag{93}$$

That means, we have to know the uncertainty of the round-trip delay ($delay_{MM}$) calculated by the WR PTP software. Table 1 presents the experimental $delay_{MM}$ uncertainties measured for two WR Switches and two SPEC cards running the White Rabbit PTP Core. The measurement was performed with a short (5 m) fiber, long (5 km) fiber and both fibers connected together (5+ km). Table shows that the standard deviation is at the picosecond level even when a single sample of $delay_{MM}$ estimation is taken instead of the mean value.

device	*fiber len.*	$\overline{del_{MM}}$ [ps]	$s^2(del_{MM})$ [ps^2]	$s^2(\overline{del_{MM}})$ [ps^2]	$s(del_{MM})$ [ps]	$s(\overline{del_{MM}})$ [ps]
WR Switch	5 m	962151	5.38	0.054	2.32	0.23
	5 km	51333653	8.53	0.085	2.92	0.29
	5+ km	51377317	8.97	0.090	2.99	0.30
WRPC / SPEC	5 m	712223	17.58	0.176	4.19	0.42
	5 km	51085804	21.31	0.21	4.61	0.46
	5+ km	51138454	23.02	0.23	4.80	0.48

Table 1: Experimental uncertainties of $delay_{MM}$ for various WR Devices based on N=100 observations made in 1s intervals

Based on those values and using equation 93 we can calculate the uncertainty of a fiber latency estimation made with two WR Switches and using a single sample or an averaged $delay_{MM}$:

$$u_c^2(\delta_1) = s^2(del_{MM5+km}) + s^2(del_{MM5km}) = 17.5[ps^2] \quad (94)$$

$$u_c^2(\delta_2) = s^2(del_{MM5+km}) + s^2(del_{MM5m}) = 14.4[ps^2] \quad (95)$$

$$u_c^2(\bar{\delta}_1) = s^2(\overline{del_{MM5+km}}) + s^2(\overline{del_{MM5km}}) = 0.175[ps^2] \quad (96)$$

$$u_c^2(\bar{\delta}_2) = s^2(\overline{del_{MM5+km}}) + s^2(\overline{del_{MM5m}}) = 0.144[ps^2] \quad (97)$$

These uncertainties are negligible for the current WR needs. However, the latency of the fiber also changes with the temperature, which may be the source of additional measurement error. Figure 10 shows how the round-trip delay changes over time for the two fiber cables: 5km and 5m.

a)

b)

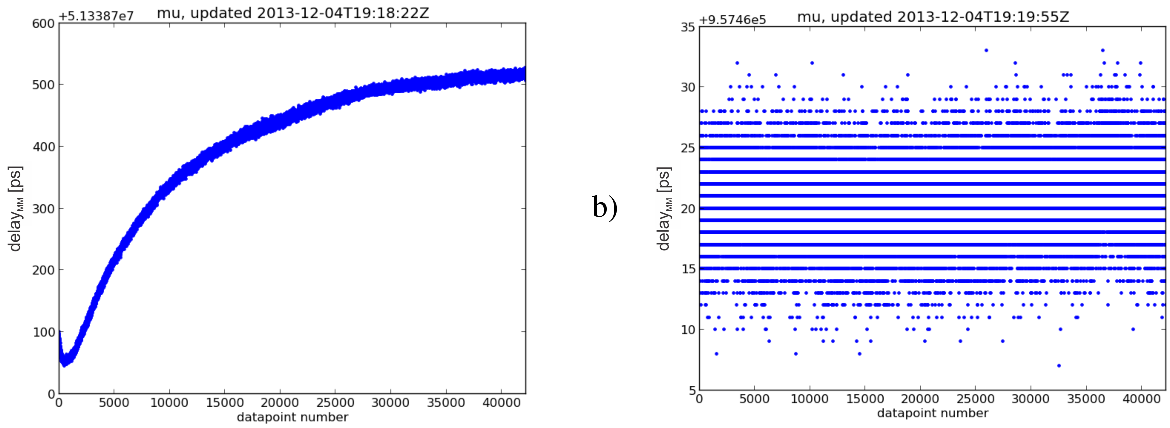

Figure 10: $delay_{MM}$ of 5 km (a) and 5 m (b) fiber logged for almost 12 hours using two WR Switches

It is noticeable, that the latency of the long fiber (5 km) varies much more over time (caused by the temperature change) than the latency of the short one. Table 2 presents the variances calculated for this almost 12-hour data set.

fiber len.	$s^2(\delta)$ [ps^2]	$s(\delta)$ [ps]
5 m	7.92	2.81
5 km	16273	128

Table 2: Long term uncertainty of $delay_{MM}$ measurement caused by temperature fluctuations

This means, the latency of the long fiber measured once, may not be valid anymore when used for the calibration procedures few days later - in a different ambient temperature. There are two conclusions from this fact:

- when you want to measure the α parameter of a long fiber, perform the latency measurement (section 4.2) prior the oscilloscope measurements (section 4.3);
- use a short (few meters) fiber to calibrate a WR Calibrator and all WR Devices (sections 4.4, 4.5), then the latency measurement of the short fiber performed only once will be accurate enough independently of the ambient temperature.

B.2 Fiber asymmetry

As described in section 4.3, the asymmetry of the fiber is expressed with the α parameter:

$$\alpha = \frac{2(skew_{PPS2} - skew_{PPS1})}{\frac{1}{2}\delta - (skew_{PPS2} - skew_{PPS1})} \tag{98}$$

The uncertainty of α depends on:

- the uncertainty of the fiber latency estimation ($u^2(\delta)$)
- the uncertainty of the 1-PPS skew between the two WR Devices ($u^2(skew_{PPS})$).

The former was already addressed in the previous section (B.1). The uncertainty of the 1-PPS skew between two WR Devices is the measuring instrument uncertainty subtracted from the uncertainty of 1-PPS skew measurement done with this instrument:

$$u^2(skew_{PPS}) = u^2(meas_{PPS}) - u^2(instr.) \tag{99}$$

The uncertainty of the instrument (e.g. oscilloscope) can be taken from its datasheet or measured in the lab. The measurement can be done by feeding the same signal to both oscilloscope channels (fig.11). You can use either a 50 Ω splitter (fig.11.1) or make a daisy-chain connection (fig.11.2). The latter requires setting one oscilloscope channel to high impedance and the other to 50 Ω termination. The signal fed to an oscilloscope can be taken from a signal generator (e.g. Agilent 33250A) or it can be also a 62.5MHz clock output taken from a WR Switch. An example measurement done this way for the *Lecroy Wavepro 7300A* oscilloscope using a 62.5MHz clock output from a free-running WR Switch is presented in figure 12. For our considerations the uncertainty of the measuring instrument is:

$$u(instr.) = 3.6[ps] \tag{100}$$

$$u^2(instr.) = 12.96[ps^2] \tag{101}$$

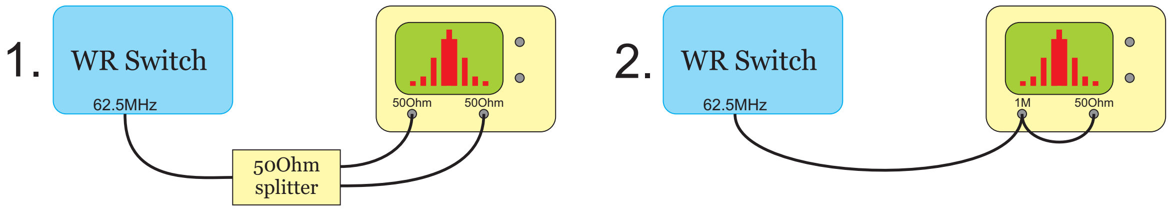

Figure 11: Measuring internal jitter of an oscilloscope

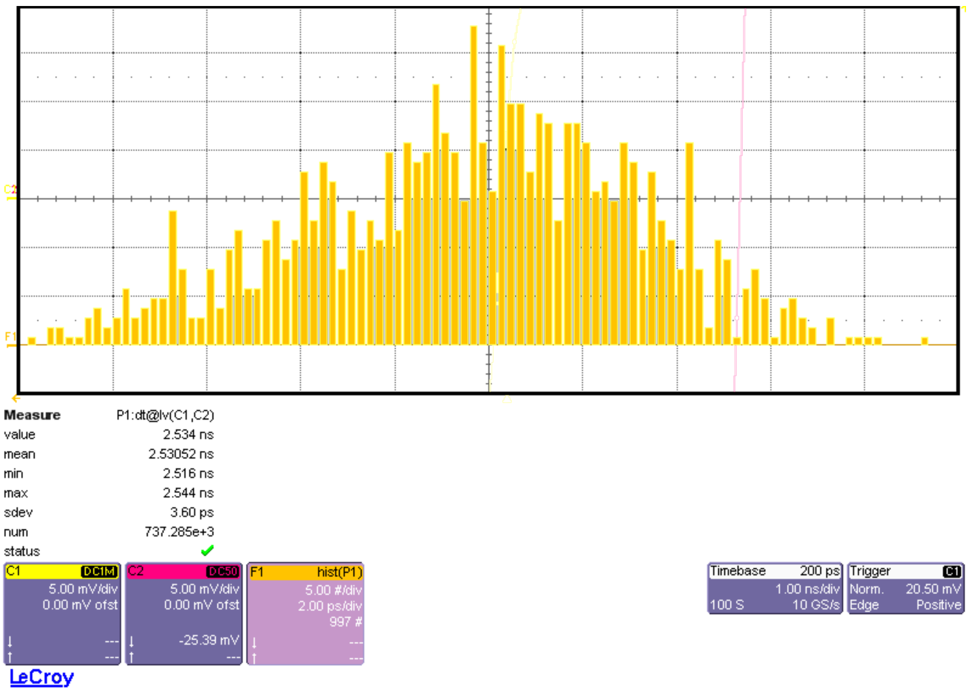

Figure 12: Uncertainty of LeCroy Wavepro 7300A oscilloscope

For comparison, the uncertainty of a *Rhode&Schwarz RTO1004* oscilloscope measured the same way is $u^2(instr.) = 30.80[ps^2]$

The next step is to determine the total uncertainty of the 1-PPS skew measurement ($u^2(meas_{PPS})$). It can be done with an oscilloscope by logging the 1-PPS skew between two synchronized WR Devices. The standard deviation and the variance can be calculated based on these samples. Parameters measured with the *Lecroy Wavepro 7300A* for two WR Switches v3.3, two SPECs running the WR PTP Core v2.1 and the WR Switch with SPEC are presented in table 3.

Using equation 99, values from 101 and table 3 the uncertainty of the 1-PPS skew between two WR Devices can be calculated. The uncertainties for various configurations are collected in table 4.

These uncertainties can be used then to calculate the uncertainty of the α parameter estimated according to step 4.3 of the calibration procedure. To simplify the equations, we intro-

devices	$u(meas_{PPS})[ps]$	$u^2(meas_{PPS})[ps^2]$
WR Switch - WR Switch	9.33	87.05
WR Switch - SPEC	19.37	375.20
SPEC - SPEC	26.79	717.70

Table 3: Total jitter of 1-PPS skew measurement

devices	$u(skew_{PPS})[ps]$	$u^2(skew_{PPS})[ps^2]$
WR Switch - WR Switch	8.61	74.09
WR Switch - SPEC	19.03	362.24
SPEC - SPEC	26.55	704.74

Table 4: Jitter of 1-PPS skew produced by various WR devices

duce a new parameter s with uncertainty $u^2(s)$:

$$s = skew_{PPS2} - skew_{PPS1} \tag{102}$$

$$u^2(s) = u^2(skew_{PPS2}) + u^2(skew_{PPS1}) = 2 \cdot u^2(skew_{PPS}) \tag{103}$$

Note: If two exactly the same WR Devices (with the same firmware version) are used for this step of the calibration, then the measured $skew_{PPS1}$ will be equal to 0 ps. That is because there won't be any asymmetry of the hardware for a given connection and the asymmetry of a few meters long fiber is negligible. In such case doing only one measurement of the 1-PPS skew ($skew_{PPS2}$) is enough and estimations s, $u^2(s)$ can be simplified:

$$s = skew_{PPS2} \tag{104}$$

$$u^2(s) = u^2(skew_{PPS}) \tag{105}$$

By putting parameter s to the α equation we get a new formula for estimating the uncertainty:

$$\alpha = \frac{2 \cdot s}{\frac{1}{2}\delta - s} \tag{106}$$

The combined standard uncertainty of the α is then estimated as:

$$\begin{aligned} u_c^2(\alpha) &= \left(\frac{\partial \alpha}{\partial s}\right)^2 u^2(s) + \left(\frac{\partial \alpha}{\partial \delta}\right)^2 u^2(\delta) \\ &= \left(\frac{\delta}{(\frac{1}{2}\delta - s)^2}\right)^2 u^2(skew_{PPS}) + \left(\frac{-s}{(\frac{1}{2}\delta - s)^2}\right)^2 u^2(\delta) \end{aligned} \tag{107}$$

Taking a look at the $\left(\frac{\partial \alpha}{\partial s}\right)$ factor, we can conclude that the influence of the 1-PPS skew uncertainty is smaller when the latency is greater (fiber is longer):

$$\lim_{\delta \to \infty} \left(\frac{\partial \alpha}{\partial s}\right) = 0 \tag{108}$$

As an example we can calculate the combined uncertainty of α using equation 107 for a 5 km fiber measured with the *LeCroy Wavepro 7300A* and two WR Switches:

$$\delta = 50421913[ps] \qquad s = 3243[ps]$$
$$u^2(\delta) = 14.4[ps^2] \qquad u^2(skew_{PPS}) = 74.09[ps^2]$$
$$\alpha = 2.573e-4 \tag{109}$$
$$u_c^2(\alpha) = 4.665e-13 \tag{110}$$
$$u_c(\alpha) = 0.0068e-4 \tag{111}$$

The uncertainty of α may cause about *9 ps* of uncompensated asymmetry in a 5km link. It can be calculated by subtracting the WR PTP $delay_{MS}$ estimations for the measured α (equation 109) and for α distorted by its standard deviation:

$$error = delay_{MS}(\alpha \pm u_c(\alpha)) - delay_{MS}(\alpha) \approx 9[ps] \tag{112}$$

B.3 Calibrator pre-calibration

The parameters of a WR Calibrator determined in the calibration procedure depend on two experimental values:

- round trip delay $delay_{MM}$
- fiber latency δ

The combined standard uncertainties of a transmission and reception delay are the same and can be estimated in a similar way as in previous sections:

$$u_c^2(\Delta_{TX}) = u_c^2(\Delta_{RX}) \tag{113}$$
$$u_c^2(\Delta_{TX}) = \left(\frac{\partial \Delta_{TX}}{\partial delay_{MM}}\right)^2 u^2(delay_{MM}) + \left(\frac{\partial \Delta_{TX}}{\partial \delta}\right)^2 u^2(\delta)$$
$$= \frac{1}{16}u^2(delay_{MM}) + \frac{1}{16}u^2(\delta) \tag{114}$$

As an example we can calculate the combined uncertainty of Δ_{TX} when port *1* of the WR Switch is selected to be a WR Calibrator and the calibration is done with the 5m fiber used in the previous steps. To evaluate the worst case, the uncertainties for single sample measurements (not means) are used here ($s^2(del_{MM})$ from table 1 and $u_c^2(\delta_1)$ from equation 94):

$$u_c^2(\Delta_{TX}) = \frac{1}{16} \cdot 5.38 + \frac{1}{16} \cdot 17.5 = 1.43[ps^2] \tag{115}$$
$$u_c(\Delta_{TX}) = 1.20[ps] \tag{116}$$

B.4 WR Device calibration

The last step of the calibration procedure depends on all the previous stages. First, we should estimate the uncertainty of the coarse (average) transmission and reception delays of a WR Device. Rewriting equation 16 and omitting the bitslide value gives us equation 117 which is used for the uncertainty estimation (118). The fixed delays of the WR Calibrator are represented by symbols Δ_{TXC} and Δ_{RXC}.

$$\Delta'_{TXS} = \Delta'_{RXS} = \frac{1}{2}(delay_{MM} - \Delta_{TXC} - \Delta_{RXC} - \delta) \tag{117}$$

$$\begin{aligned} u^2(\Delta'_{TXS}) &= \left(\frac{\partial \Delta'_{TXS}}{\partial delay_{MM}}\right)^2 u^2(delay_{MM}) + 2\cdot\left(\frac{\partial \Delta'_{TXS}}{\partial \Delta_{TXC}}\right)^2 u^2(\Delta_{TXC}) + \left(\frac{\partial \Delta'_{TXS}}{\partial \delta}\right)^2 u^2(\delta) \\ &= \frac{1}{4}u^2(delay_{MM}) + \frac{1}{2}u^2(\Delta_{TXC}) + \frac{1}{4}u^2(\delta) \end{aligned} \tag{118}$$

For the case when the WR Switch is the calibrator, the WRPC running on a SPEC is the WR Device under calibration and the connection is made with a short fiber, the uncertainty of the coarse fixed delay is:

$$u^2(\Delta'_{TXS}) = u^2(\Delta'_{RXS}) = \frac{1}{4}\cdot 17.58 + \frac{1}{2}\cdot 1.43 + \frac{1}{4}\cdot 17.5 = 9.49[ps^2] \tag{119}$$

$$u(\Delta'_{TXS}) = u(\Delta'_{RXS}) = 3.08[ps] \tag{120}$$

The second part of the WR Device calibration uses the 1-PPS skew readout from the oscilloscope as the correction value β to compensate fixed delay asymmetry. The uncertainty of the estimation of β is then the sum of two factors: the uncertainty of the 1-PPS skew measurement (the same as in B.2) and the uncertainty of the one way delay ($delay_{MS}$) estimation in the WR PTP software:

$$u^2(\beta) = u^2(skew_{PPS}) + u^2(delay_{MS}) \tag{121}$$

Knowing the formula used by the WR PTP software for estimating $delay_{MS}$:

$$delay_{MS} = \frac{1+\alpha}{2+\alpha}(delay_{MM} - \Delta) + \Delta_{TXM} + \Delta_{RXS} \tag{122}$$

we can calculate the uncertainty of this estimation:

$$\begin{aligned} u^2(delay_{MS}) = &\left(\frac{\partial delay_{MS}}{\partial \alpha}\right)^2 u^2(\alpha) + \left(\frac{\partial delay_{MS}}{\partial delay_{MM}}\right)^2 u^2(delay_{MM}) \\ &+ \left(\frac{\partial delay_{MS}}{\partial \Delta}\right)^2 u^2(\Delta) + \left(\frac{\partial delay_{MS}}{\partial \Delta_{TXC}}\right)^2 u^2(\Delta_{TXC}) \\ &+ \left(\frac{\partial delay_{MS}}{\partial \Delta_{RXS}}\right)^2 u^2(\Delta'_{RXS}) \end{aligned} \tag{123}$$

All the necessary uncertainties but $u^2(\Delta)$ were calculated in previous sections. However, Δ is the sum of the coarse fixed delays of the Master (Calibrator) and the Slave, so:

$$u^2(\Delta) = 2\cdot u^2(\Delta_{TXC}) + 2\cdot u^2(\Delta'_{RXS}) \tag{124}$$

$$= 2\cdot 1.43 + 2\cdot 9.49 = 21.84[ps^2] \tag{125}$$

Table 5 presents the values from the SPEC calibration (the WR Switch was the Calibrator) and collects together other uncertainties calculated in previous sections. Please note that the a)

$delay_{MM}$	Δ_{TXC}	Δ_{RXC}	Δ_{TXS}	Δ_{RXS}	α
838152	225030	228230	164261	170661	2.573e-4

b)

$u^2(\alpha)$	$u^2(delay_{MM})[ps^2]$	$u^2(\Delta)[ps^2]$	$u^2(\Delta_{TXC})[ps^2]$	$u^2(\Delta'_{RXS})[ps^2]$
4.665e-13	17.58	21.84	1.43	9.49

Table 5: Example of SPEC calibration values when being calibrated to a WR Switch (a) and uncertainties calculated in previous sections (b)

transmission and reception fixed delays are not equal because of the bitslide. They could have been omitted earlier, but now have to be taken into account for Δ calculation.

Using these values we can calculate the uncertainty of the estimation of $delay_{MS}$ (126).

$$u^2(delay_{MS}) = \left(\frac{delay_{MM} - \Delta}{(2+\alpha)^2}\right)^2 u^2(\alpha) + \left(\frac{1+\alpha}{2+\alpha}\right)^2 u^2(delay_{MM}) + \left(-\frac{1+\alpha}{2+\alpha}\right)^2 u^2(\Delta) + u^2(\Delta_{TXC}) + u^2(\Delta'_{RXS}) \tag{126}$$

$$u^2(delay_{MS}) = 20.78[ps^2] \tag{127}$$

$$u(delay_{MS}) = 4.56[ps] \tag{128}$$

Taking the uncertainty of the 1-PPS skew produced by the SPEC and the WR Switch (table 4) and comparing it to the uncertainty of the estimation of $delay_{MS}$ and the uncertainty of the measuring instrument (equation 101) we can conclude that almost all of the uncertainty comes from the jitter of the 1-PPS output from the SPEC:

$$u^2(\beta) = u^2(skew_{PPS}) + u^2(delay_{MS}) \approx u^2(skew_{PPS}) \approx 362[ps^2] \tag{129}$$

$$u(\beta) \approx 19[ps] \tag{130}$$

References

[1] J.Serrano P.Alvarez J.Lewis D.Autiero. Inter-laboratory synchronization for the cngs project. *10th European Particle Accelerator Conference, Edinburgh, UK, 26 - 30 Jun 2006, pp.3092*.

[2] Working Group 1 of the Joint Committee for Guides in Metrology(JCGM/WG 1). *Evaluation of measurement data - Guide to the expression of uncertainty in measurement (JCGM 100:2008)*, September 2008.