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IEEE1588 DELAY ASYMMETRY MEASUREMENT MODE: THREE ONE-WAY DELAY MATH

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THREE ONE-WAY-DELAYS MATH

IEEE1588-2019 **7.4.2** *), **):

[1] $$< meanPathDelay > = \frac{t_{ms}+t_{sm}}{2}$$

[2] $$t_{ms} = < meanPathDelay > + < delayAsymmetry >$$

[3] $$t_{sm} = < meanPathDelay > - < delayAsymmetry >$$

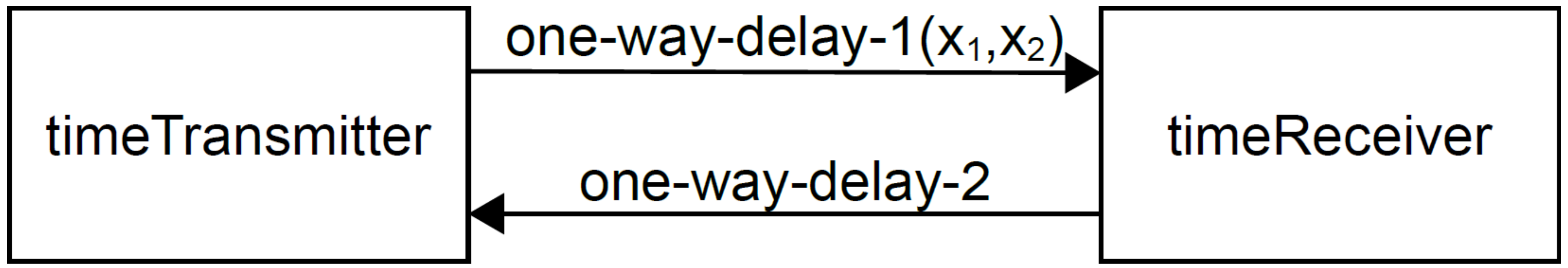
From 2 and 3 follows:

[4] $$< delayAsymmetry > = \frac{t_{ms}-t_{sm}}{2}$$

Suppose **timeTransmitter to timeReceiver** changes transmission characteristic x then

t_{ms} is a function of x_{ms}

t_{sm} is constant



$< roundTripDelay >$ as a function of x_{ms} is:

[5] $$< \mathrm{roundTripDelay}(x_{ms}) > = t_{ms}(x_{ms}) + t_{sm}$$

*) kept the 2019 notation for t_{ms} and t_{sm} for the time being. Replacement of master/slave into timeTransmitter and timeReceiver would probably yield t_{tTtR} and t_{tRtT} or t_{TR} and t_{RT} respectively.
**) <meanPathDelay>, t_{ms}, t_{sm} are used here, which may be replaced by <meanLinkDelay>, $t_{resp\text{-}to\text{-}req}$, $t_{req\text{-}to\text{-}resp}$ when peer-to-peer delay mechanism is used.

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Substitute [5] into [4] :

[6] $$< \text{delayAsymmetry}(x_{ms}) > = \frac{<\text{roundTripDelay}(x_{ms})> - 2t_{sm}}{2}$$

At equal transmission characteristic $x_{ms} = x_{sm}$, $< \text{delayAsymmetry} > = 0$ so [6] results in:

[7] $$2t_{sm} = < \text{roundTripDelay}(x_{sm}) >$$

Substitute [7] into [6] leads to $< \text{delayAsymmetry} >$ as a function of x in terms of round-trip-delays:

[8] $$< \text{delayAsymmetry}(x_{ms}) > = \frac{<\text{roundTripDelay}(x_{ms})> - <\text{roundTripDelay}(x_{sm})>}{2}$$

For $x_{ms} = x_1, x_2$ and fixed x_{sm}, t_{ms} can be expressed as t_{sm} plus a time t depending on x_{ms}

[9] $$t_{ms}(x_{ms}) = t_{sm} + t(x_{ms})$$

From [5] follows

[10] $$< \text{roundTripDelay}(x_{ms}) > = t(x_{ms}) + 2t_{sm}$$

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Substitute [10] into [6] :

[11] $$< \text{delayAsymmetry}(x_{ms}) > = \frac{t(x_{ms})}{2}$$

Under the assumption that there is a (nearly) linear relation of one-way-delay as a function of transmission characteristic x, for (small) Δx, scaling [11] to Δx yields a tangent slope in ***[seconds per "unit of x"]***:

[12] $$\frac{<\text{delayAsymmetry}(x_{ms})>}{\Delta_x} = \frac{t(x_{ms})}{2\Delta_x} = tangent$$

define

[13a] $$\Delta x_1 = x_1 - x_{sm}$$

[13b] $$\Delta x_2 = x_2 - x_{sm}$$

combine [12] and [13]

[14] $$\frac{<\text{delayAsymmetry}(x_1)>}{\Delta x_1} = \frac{<\text{delayAsymmetry}(x_2)>}{\Delta x_2} = \frac{t(x_1)}{2\Delta x_1} = \frac{t(x_2)}{2\Delta x_2}$$

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thus:

[15] $$t(x_2) = t(x_1)\frac{\Delta x_2}{\Delta x_1}$$

measured $< \mathrm{roundTripDelay} >$ for x_1 and $x_2$can be expressed using [10]

[16] $$< \mathrm{roundTripDelay}(x_1) > \; - < \mathrm{roundTripDelay}(x_2) > = t(x_1) - t(x_2)$$

feeding [15] into [16]

[17] $$t(x_1) = (< \mathrm{roundTripDelay}(x_1) > \; - < \mathrm{roundTripDelay}(x_2) >)\frac{\Delta x_1}{\Delta x_1 - \Delta x_2}$$

$< \mathrm{delayAsymmetry} >$ as a function of measured $< \mathrm{roundTripDelay} >$ at x_1 and x_2 is (substitute [17] into [11]):

[18] $$< \mathrm{delayAsymmetry}(x_1) > =$$

$$(< \mathrm{roundTripDelay}(x_1) > \; - < \mathrm{roundTripDelay}(x_2) >)\frac{\Delta x_1}{2(\Delta x_1 - \Delta x_2)}$$

THREE ONE-WAY-DELAYS MATH

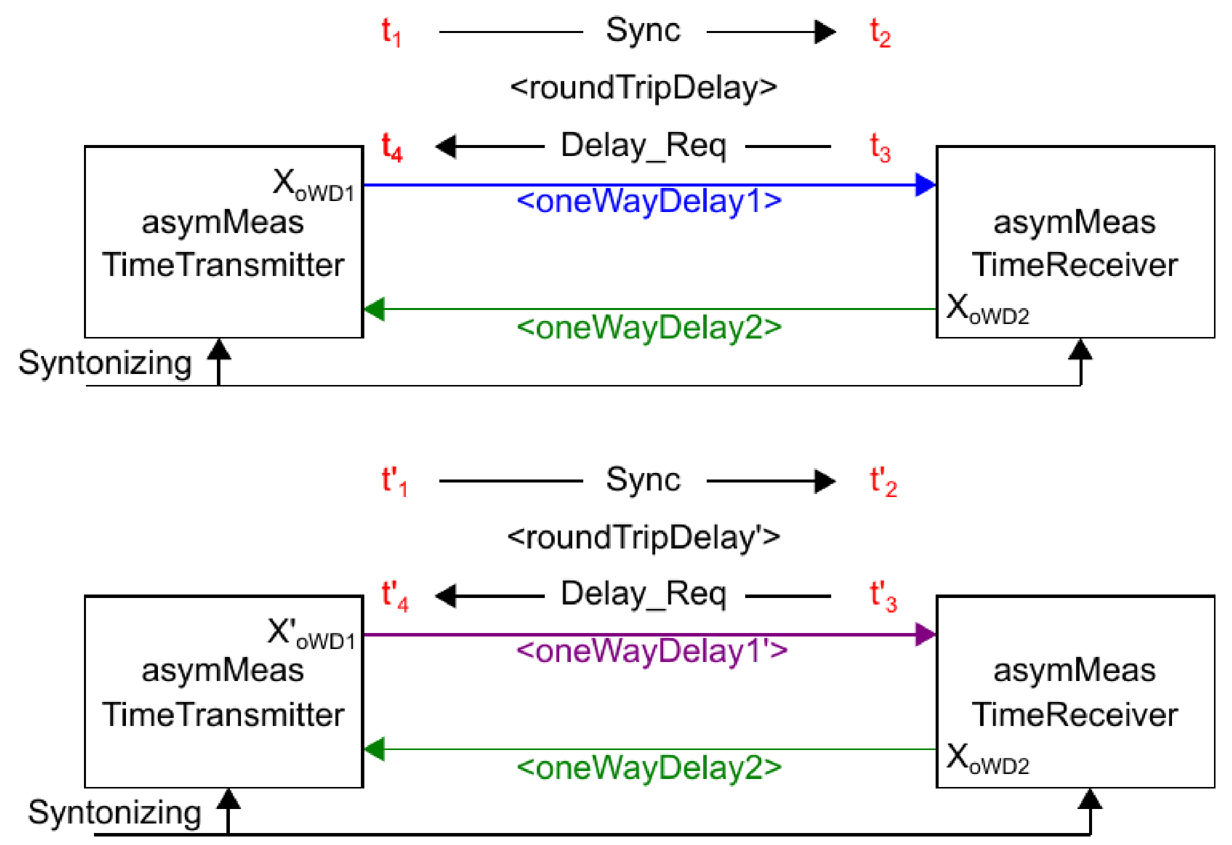
substitute Δx_1, Δx_2 as defined in [13]:

[19] $$< \mathrm{delayAsymmetry}(x_1) > = \frac{(x_1 - x_{sm})(<\mathrm{roundTripDelay}(x_1) - <\mathrm{roundTripDelay}(x_2))}{2(x_1 - x_2)}$$

In IEEE 1588 nomenclature:

x_1 = X_{oWD1}
x_2 = X'_{oWD1}
x_{sm} = X_{oWD2}
$< \mathrm{roundTripDelay}(x_1) >$ = <roundTripDelay>
$< \mathrm{roundTripDelay}(x_2) >$ = <roundTripDelay’>

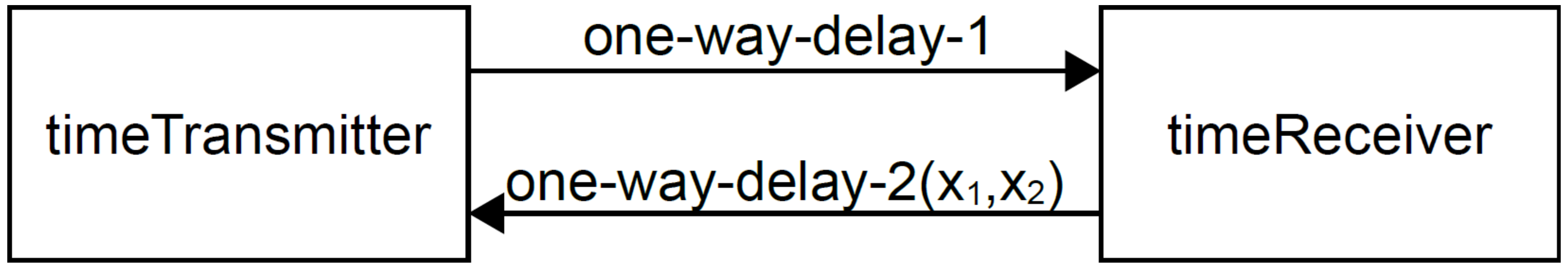
a) Constant transmission characteristic X_{oWD2}



[20] $$< \mathrm{delayAsymmetry} > = \frac{(X_{oWD1} - X_{oWD2})(<\mathrm{roundTripDelay}> - <\mathrm{roundTripDelay'}>)}{2 \,.\, (X_{oWD1} - X'_{oWD1})}$$

THREE ONE-WAY-DELAYS MATH

The other way around:
Suppose **timeReceiver to timeTransmitter** changes transmission characteristic then
t_{ms} is constant
t_{sm} is a function of x_{sm}



THREE ONE-WAY-DELAYS MATH

$< roundTripDelay >$ as a function of x_{sm} is:

[5’] $< \mathrm{roundTripDelay}(x_{sm}) > = t_{ms} + t_{sm}(x_{sm})$

Substitute [5’] into [4] :

[6’] $< \mathrm{delayAsymmetry}(x_{sm}) > = \frac{2t_{ms} - <\mathrm{roundTripDelay}(x_{sm})>}{2}$

At equal transmission characteristic $x_{sm} = x_{ms}$, $< \mathrm{delayAsymmetry} > = 0$ so [6’] results in:

[7’] $2t_{ms} = < \mathrm{roundTripDelay}(x_{ms}) >$

Substitute [7’] into [6’] leads to $< \mathrm{delayAsymmetry} >$ as a function of x in terms of round-trip-delay:

[8’] $< \mathrm{delayAsymmetry}(x_{sm}) > = \frac{<\mathrm{roundTripDelay}(x_{ms})> - <\mathrm{roundTripDelay}(x_{sm})>}{2}$

***) Note: kept the 2019 notation for t_{ms} and t_{sm} for the time being. Replacement of master/slave into timeTransmitter and timeReceiver would probably yield t_{tTtR} and t_{tRtT} or t_{TR} and t_{RT} respectively**

THREE ONE-WAY-DELAYS MATH

For $x_{sm} = x_1, x_2$ and fixed x_{ms}, t_{sm} can be expressed as t_{ms} plus a time t depending on x_{sm}

[9’] $$t_{sm}(x_{sm}) = t_{ms} + t(x_{sm})$$

From [5’] follows

[10’] $$< \mathrm{roundTripDelay}(x_{sm}) > = 2t_{ms} + t(x_{sm})$$

Substitute [10’] into [6’] :

[11’] $$< \mathrm{delayAsymmetry}(x_{sm}) > = \frac{-t(x_{sm})}{2}$$

Under the assumption that there is a (nearly) linear relation of one-way-delay as a function of transmission characteristic x, for (small) Δx, scaling [11’] to Δx yields a tangent slope in ***[seconds per “unit of x”]***:

[12’] $$\frac{<\mathrm{delayAsymmetry}(x_{sm})>}{\Delta_x} = \frac{-t(x_{sm})}{2\Delta_x} = tangent$$

THREE ONE-WAY-DELAYS MATH

define

[13a'] $\Delta x_1 = x_1 - x_{ms}$

[13b'] $\Delta x_2 = x_2 - x_{ms}$

combine [12'] and [13']

[14'] $\frac{<\text{delayAsymmetry}(x_1)>}{\Delta x_1} = \frac{<\text{delayAsymmetry}(x_2)>}{\Delta x_2} = \frac{-t(x_1)}{2\Delta x_1} = \frac{-t(x_2)}{2\Delta x_2}$

thus:

[15'=15] $t(x_2) = t(x_1)\frac{\Delta x_2}{\Delta x_1}$

measured $< \text{roundTripDelay} >$ for x_1 and x_2 can be expressed using [10']

[16'=16] $< \text{roundTripDelay}(x_1) > - < \text{roundTripDelay}(x_2) > = t(x_1) - t(x_2)$

feeding [15'] into [16']

[17'=17] $t(x_1) = (< \text{roundTripDelay}(x_1) > - < \text{roundTripDelay}(x_2) >)\frac{\Delta x_1}{\Delta x_1 - \Delta x_2}$

THREE ONE-WAY-DELAYS MATH

$< \text{delayAsymmetry} >$ as a function of measured $< \text{roundTripDelay} >$ at x_1 and x_2 is (substitute [17'] into [11']):

[18'] $$< \text{delayAsymmetry}(x_1) > =$$

$$-(< \text{roundTripDelay}(x_1) > - < \text{roundTripDelay}(x_2) >) \frac{\Delta x_1}{2(\Delta x_1 - \Delta x_2)} =$$

THREE ONE-WAY-DELAYS MATH

substitute Δx_1, Δx_2 as defined in [13']:

$$[19'] \qquad < \mathrm{delayAsymmetry}(x_1) > = \frac{-(x_1 - x_{ms})(<\mathrm{roundTripDelay}(x_1)> - <\mathrm{roundTripDelay}(x_2)>)}{2(x_1 - x_2)}$$

In IEEE 1588 nomenclature:

$x_{ms} = X_{oWD1}$

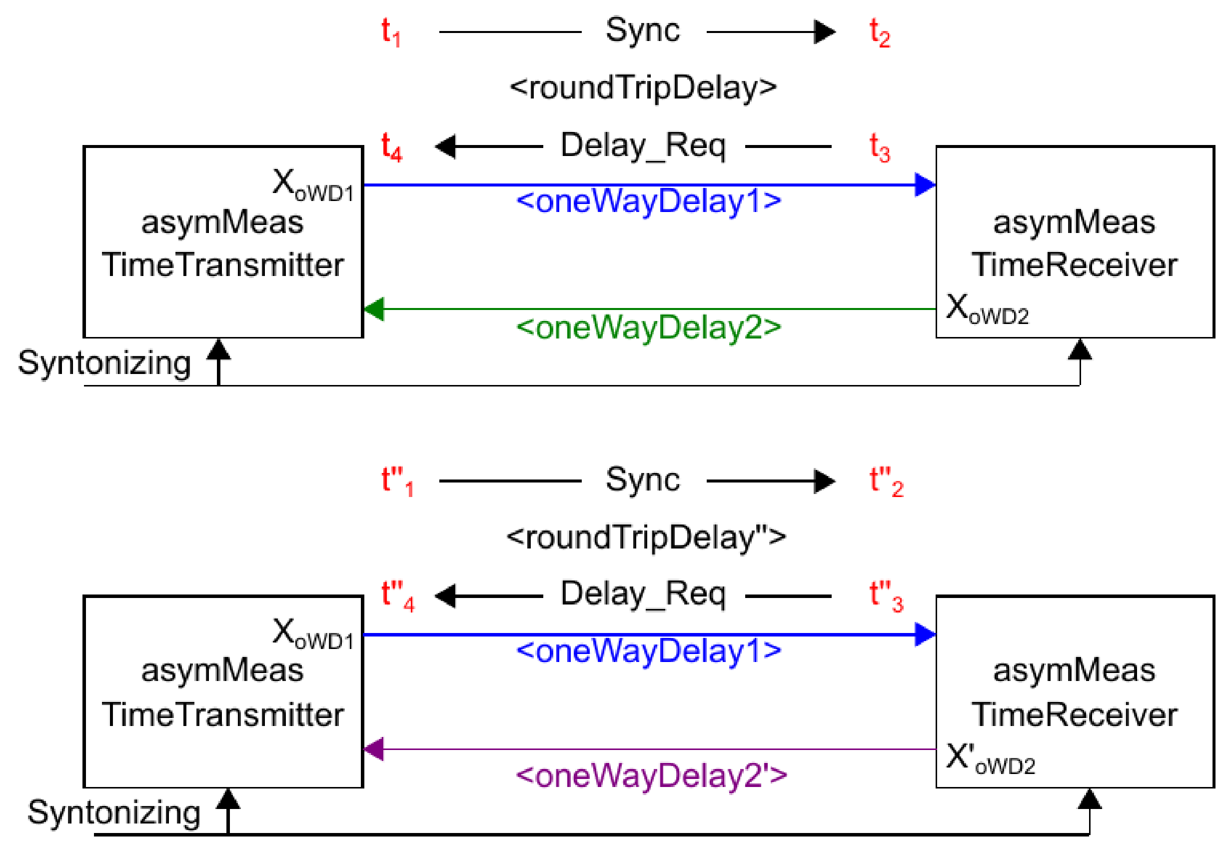
$x_1 = X_{oWD2}$

$x_2 = X'_{oWD2}$

$< \mathrm{roundTripDelay}(x_1) >$ = <roundTripDelay>

$< \mathrm{roundTripDelay}(x_2) >$ = <roundTripDelay''>

b) Constant transmission characteristic X_{oWD1}



$$[20'] \qquad < \mathrm{delayAsymmetry} > = \frac{(X_{oWD1} - X_{oWD2})(<\mathrm{roundTripDelay}> - <\mathrm{roundTripDelay''}>)}{2 \,.\, (X_{oWD2} - X'_{oWD2})}$$